	COOMET Recommendation	**COOMET**
	Guideline on COOMET supplementary comparison evaluation	**R/GM/19:2016**

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1. Introduction

The supplementary comparisons for national standards are carried out with the purpose of confirming calibration and measurement capabilities (CMC) [1] of the corresponding national metrology institutes (NMI).

During the evaluation of the supplementary comparisons data the measurement uncertainties claimed by the participants of comparisons are confirmed, that, essentially, is the confirmation of corresponding measurement capabilities. Rather frequently the measurement procedures during the comparisons are the procedures of calibration in these laboratories. In such a case, one might say about the confirmation of calibration capabilities of the laboratory also. If the calibration procedures differ from the measurement procedures during the comparisons, then the comparisons confirm only the certain part of the calibration measurement uncertainty. In such a case the procedure of the confirming of calibration capabilities requires additionally the analysis of the quality management system, including the examination of the corresponding calibration procedure.

2. Conditions of use

The procedures offered in this *Guideline* are applicable for the evaluation of the supplementary comparisons data, if the following conditions are fulfilled:

1. The traveling standard is stable.
2. Each national institute that participates in supplementary COOMET comparisons provides the measurement result and corresponding combined standard uncertainty and uncertainty budget.
3. For each national institute, that participates in additional COOMET comparisons, the Gaussian distribution can be assigned to the measurand on the basis of the available information.

3. Foundation

It is rational to define two types of supplementary comparisons concerning the data evaluation.

(1) Type 1. The comparisons of the primary national standards, that for some different reasons were not registered as key comparisons. Among such reasons one might mark the overload of the Consultative Committees and the interest in the given sort of comparison of only limited number of NMIs. The comparisons of that type can often be bilateral.

The measurement procedure of the supplementary comparisons of the first type practically doesn't differ from the key comparison procedure. The pilot laboratory sends the traveling standard, participants provide the results with corresponding measurement uncertainties and the uncertainty budget. The circular, radial or hybrid scheme of comparisons is defined by the traveling standard properties, first of all by its stability. On the basis of the measurement results, claimed by the participants, the reference value of the supplementary comparisons is calculated. Some differences of the data processing of the given type

comparisons from the CIPM key comparisons are determined by the interpretation of the comparison results, because the degrees of equivalence between standards are not established.

(2) Type II. Generally, the comparisons of the secondary national standards, that take the size of unit from the participants of the key comparisons. The participation of such institutes in the CIPM KC is problematic, because the group of the CIPM KC participants is, first of all, limited by primary national standards, that have approximately the same level of accuracy, and the measurement methods used during the key comparison realization. For realization of the supplementary comparisons of that type it is necessary to engage in the reference laboratory, that have been the participant of the key comparisons in given area of measurements. The purpose of this laboratory is to determine the reference value of the supplementary comparisons. It is recommended to distinguish the terms "the comparisons reference value" and "the supplementary comparisons reference value". Generally the supplementary comparisons of the second type are carried out strictly to confirm CMC, that's why in this case the measurement procedure during the comparisons and the calibration procedure are identical. During the evaluation of this type comparisons data it is important to take into account the correlation of measurement results due to taking the size of unit.

4. Notation used

- x_i - results of the supplementary comparisons;
- $u(x_i)$ - quoted standard uncertainties of the participants of comparisons
- n –the number of the participants of comparisons
- x_{ref} -the reference value of the additional comparisons
- $u(x_{ref})$ - standard uncertainty of the reference value of the additional comparisons
- $\mathrm{cov}(x_i, x_{ref})$ - covariation of the *i*- th NMI measurement result and the reference value of the additional comparisons
- CMC - calibration and measurement capabilities

5. The evaluation of the supplementary comparisons data. Type I.

5.1 Checking the comparisons data consistency

5.1.1 On the basis of the measurement results and corresponding uncertainties $\{x_i, u(x_i)\}, i = 1,...,n$, claimed by comparisons participants, the χ^2 criterion value is calculated

$$\chi^2 = \sum_1^n \frac{(x_i - x_{ref})^2}{u^2(x_i)} \tag{1}$$

where

$$x_{ref} = \frac{\sum_1^n \frac{x_i}{u^2(x_i)}}{\sum_1^n \frac{1}{u^2(x_i)}}, \quad u^2(x_{ref}) = \frac{1}{\sum_1^n \frac{1}{u^2(x_i)}} \tag{2}$$

5.1.2. If the criterion value calculated in accordance with the data provided by NMIs doesn't exceed the critical value χ^2 with the coverage level 0,95 and the degrees of freedom *n-1*

$$\chi^2 = \sum_1^n \frac{(x_i - x_{ref})^2}{u^2(x_i)} \quad \prec \quad \chi^2_{0,95}(n-1), \tag{3}$$

then the data provided by different NMIs can be acknowledged as consistent, that is the objective confirmation of the announced uncertainties.

In that case to confirm CMC go to 5.4.1.

5.2 The detection of inconsistent data and forming consistent set of comparisons data.

NMI that provides maximum E_n criterion is determined

$$\max_i E_n = \frac{|x_i - x_{ref}|}{2\sqrt{u^2(x_i) - u^2(x_{ref})}} \tag{4}$$

Further that NMI's data is temporary excluded from the consideration, and the procedure from clause 5.1. is repeated. The sequential exclusion of data is repeated until the condition (3) is fulfilled for the group of the remaining data (consistent data set).

To form the set of consistent data the method of forming the largest consistent set can be used also [2].

5.3 The analysis of inconsistent data

Those NMIs, whose results were excluded, have to analyze the reasons of their results falling out. As a result of analysis the following could be established:

- the measurement result is an error and NMI decides to exclude its result. In such a case the announced uncertainties doesn't confirmed during the given supplementary comparisons and for their confirmation it is necessary to participate in other similar comparisons
- NMI reveals the reasons of the understatement of the announced uncertainty assessment and provides the pilot laboratory and other comparisons participants with those reasons. The comparisons participants agree with the explanations. After that the primarily announced uncertainty is extended so that one can consider that corresponding CMCs are confirmed by the given supplementary comparisons (see 5.4.2).

5.4 Confirmation of CMC data

5.4.1. For the measurement results included in consistent data set and used for calculation of the reference value the next method of CMC data confirmation is applied.

if for the measurement result $x, u(x)$ the condition:

$$E_n = \frac{|x - x_{ref}|}{2\sqrt{u^2(x) - u^2(x_{ref})}} < 1, \tag{5}$$

is fulfilled, then the minimum standard uncertainty, that can be claimed as CMC, is:

$$u(cmc) = u(x) \tag{6},$$

Correspondingly, the extended uncertainty is $U_{0,95}(cmc) = 2u(x)$.

If for the measurement result $x, u(x)$ condition (5) is not fulfilled, then minimum standard uncertainty, that can be claimed as CMC [3], is calculated according to the next formula (7):

$$u^2(cmc) = \frac{(x - x_{ref})^2}{4} + u^2(\bar{x}_{ref}) \quad (7)$$

Correspondingly, the extended uncertainty is $U_{0,95}(cmc) = 2u(cmc)$.

5.4.2 For the measurement results not included in consistent data set and not used for calculation of the reference value, the minimum standard uncertainty, that can be claimed as CMC[3] is calculated according to the formula (8):

$$u^2(cmc) = \frac{(x - x_{ref})^2}{4} - u^2(\bar{x}_{ref}) \quad (8)$$

Correspondingly, the extended uncertainty is $U_{0,95}(cmc) = 2u(cmc)$.

6. The evaluation of the additional comparisons data. Type II.

6.1 The establishment of the additional comparisons reference value

The reference laboratory on the basis of measurement results or calculations establishes the supplementary comparisons reference value – the traveling standard value assessment and corresponding uncertainty:

$$x_{ref}, \quad u(x_{ref}) \quad (9)$$

6.2 Checking the comparisons data consistency

6.2.1. The criterion $\tilde{E}_n$ (10) is used for the confirmation of CMC. $\tilde{E}_n$ value is calculated from the result provided by *i*-th NMI and associated uncertainty $x, u(x)$:

$$\tilde{E}_n = \frac{|x - x_{ref}|}{2\sqrt{u^2(x) + u^2(x_{ref}) - 2\,\mathrm{cov}(x, x_{ref})}} < 1 \quad (10)$$

6.3 The evaluation of the covariance between measurement results and the reference value.

The basic reason for the covariance $\mathrm{cov}(x_i, x_{ref})$ is traceability of the i-th laboratory to the reference laboratory. For the evaluation of the covariance it is necessary to analyze the uncertainty budget of the comparisons participant and the reference laboratory and to reveal those components, that are common and invariable $u_0^2(x_i)$:

$$\mathrm{cov}(x_i, x_{ref}) = u_0^2(x_i) \quad (11)$$

6.2.2 If condition (11) is fulfilled, then minimum standard uncertainty, that can be claimed as CMC [3] is $u(cmc) = u(x)$. Accordingly the extended uncertainty is $U_{0,95}(cmc) = 2u(cmc)$.

6.2.3 Those NMI results, that don't comply inequality (10), do not confirm announced uncertainties. These NMIs have to carry out the analysis of their results fallout reasons. As a result of analysis the following could be established:

- the measurement result is an error and NMI decides to exclude its result. In such a case the announced uncertainties doesn't confirmed during the given additional comparisons and for their confirmation it is necessary to participate in other similar comparisons
- NMI reveals the reasons of the understatement of the announced uncertainty assessment and provides the pilot laboratory and other comparisons participants with those reasons. The comparisons participants agree with the explanations. After that the primarily announced uncertainty is extended. The

minimum standard uncertainty, that can be claimed as CMC[3] is calculated according to the formula (12):

$$u^2(cmc) = \frac{\left(x - x_{ref}\right)^2}{4} - u^2\left(x_{ref}\right) \qquad (12)$$

Accordingly the extended uncertainty is $U_{0,95}(cmc) = 2\,u\left(cmc\right)$.

References

1. Bureau International des Poids et Mesures (BIPM) 1999 Mutual recognition of national standards and of calibration and measurement certificates issued by national metrology institutes *BIPM Publication* (Sevres:BIPM)
2. Cox M. G. "The evaluation of key comparison data: determining the largest consistent subset", 2007, *Metrologia* 44, pp. 187-200
3. Cox M G,, Harris P, Milton M "Methods for determining acceptable CMC" http://www.bipm.org/wg/AllowedDocuments.jsp?wg=GAWG